First Mid term examination -

Sub - Modern control theory

Qus. 1. Explain the following :-

(a) Difference between Modern control theory & conventional control theory & also State variable and state Vector

(b) Write state equations for the network shown below.

Ans.

(a)

Modern Control theory	conventional control theory
1. This can be implemented to Linear + non-Linear System.	1. This can be implemented to only Linear System.
2. Modern control theory is based on the concept of State	2. It is based on transfer fun technique.
3. It is useful for complex tasks with good accuracy.	3. It is Less accurate
4. This is applicable to MIMO system.	4. This is applicable only to SISO system.
5. It also provides information about the internal State of the System.	5. It does not provide any information about the internal State of the System.
6. It is easier to apply where the laplace transform can not be applied.	6. It can not be applied where Laplace transform is not applicable.

State variable -

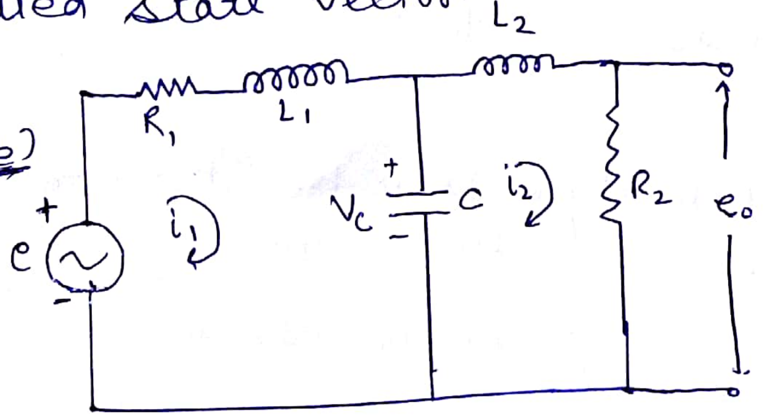
The state variables of a dynamic system are the minimal set of variables which determine the dynamics of a linear system.

→ In state variable formation of a system, the state variable are usually reprented by $x_1(t), x_2(t)$ --- & $x_1(t_0), x_2(t_0)$ --- $x_n(t_0)$ describe the initial state of the system at time $t = t_0$.

State Vector →

"If n state variables are necessary to determine the behaviour of a given system, the variabl can be considered as n components of a vector called state vector.

Sol^n (b)



$$e_o = i_2 R_2$$

Apply KVL in Loop-1

$$e = R_1 i_1 + L_1 \frac{di_1}{dt} + V_c$$

$$\frac{di_1}{dt} = -\frac{R_1}{L_1} i_1 - \frac{V_c}{L_1} + \frac{e}{L_1} \quad \text{--- (1)}$$

Apply KVL in loop-II

$$V_c = L_2 \frac{di_2}{dt} + R_2 i_2$$

$$\frac{di_2}{dt} = -\frac{R_2}{L_2} i_2 + \frac{V_c}{L_2} \quad \text{--- (2)}$$

Scanned by CamScanner

$$C\frac{dV_c}{dt} = i_1 - i_2 \Rightarrow \frac{dV_c}{dt} = \frac{i_1 - i_2}{C} \quad \text{--- (3)}$$

Now state equations

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} e$$

and,

$$e_o = \begin{bmatrix} 0 & R_2 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix}$$

Ques. 2.

(a) Derive the transfer fun from state Model

(b) Find out the transfer fun of

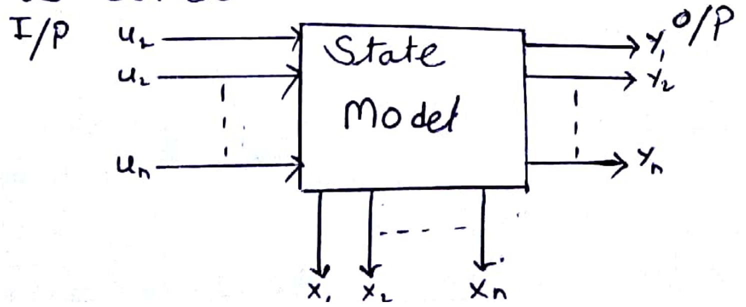
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Soln

(a) Transfer fun of state Model

Let us consider an state Model shown below



A mathematical abstraction to represent or Model the dynamics of a system utilizes three types of Variables called the input, the O/P & the State variable.

State Model -

$\dot{x}(t) = A x(t) + B u(t)$ ---- state eqn

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = [A] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + [B] \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

$y(t) = x(t) + D u(t)$ ---- o/p equation

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = [C] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + [D] \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

both equations together is called state Model

where

$x(t)$ - state vector

A - system matrix

$u(t)$ - input vector

B - input matrix

$y(t)$ - output vector

C - output Matrix

D - Transmission matrix

Applying Laplace transform to state equation

$sX(s) - X(0) = AX(s) + BU(s)$

$\Rightarrow X(s)[sI - A] = BU(s) + X(0)$

For transfer funⁿ initial condition taken zero

$$X(0) = 0$$

$$\Rightarrow X(s) = [sI - A]^{-1} B U(s) \quad \text{--- (1)}$$

Now take laplace transform of o/p eqⁿ

$$Y(s) = C X(s) + D U(s)$$

put value of $X(s)$ from eqⁿ (1)

$$Y(s) = C[sI - A]^{-1} B U(s) + D U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = T(s) = C[sI - A]^{-1} B + D}$$

Ans(b): Given.

$$A = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$C = [1 \quad 2]$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 5 & -1 \\ 3 & -1 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}$$

Now taking inverse

$$[sI - A]^{-1} = \frac{1}{s^2+6s+8} \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}$$

Now $[sI - A]^{-1} B = \frac{1}{s^2+6s+8} \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$$= \frac{1}{s^2+6s+8} \begin{bmatrix} 2s & -3 \\ 5s+31 \end{bmatrix}$$

Now T.F.

$$T(s) = C(sI-A]^{-1}B$$

$$= \frac{1}{s^2+6s+8} [1 \quad 2] \begin{bmatrix} 2s-3 \\ 5s+31 \end{bmatrix}$$

$$T(s) = \frac{12s+59}{s^2+6s+8}$$